The references cited during the exposition show the participation of the members of our group in the study of the proposed topics. Despite we have almost always chosen recent references, several members of the group developed a long career in the field of partial differential equations and harmonic analysis.

The research project we propose has three different and complementary aspects:

(A) Mathematical Physics,
(B) Mathematical Analysis and Partial Differential Equations,
(C) Numerical Analysis.

We have followed this scheme in the last six years and we still think that it is the appropriate one.

In the Mathematical Physics side (that is, **(A)**) we have three new members in the group with respect to our previous proposal: J.B. Bru and P. Zhu that have been hired by Ikerbasque, and C. Cuesta that has a Ramon y Cajal fellowship. C. Cuesta has been collaborating with J.J.L. Velazquez for many years, the same as M. Escobedo.

So we have decided to include him as an external member of the group. We are also very optimistic about the possible new interactions of Cuesta and Velazquez and our group in the field of Fluid Mechanics. More concretely in the evolution of vortex filaments using the so-called Binormal Flow (BF). Some first small steps in this interaction were already given when S. Gutiérrez had a postdoctoral position at the U. Complutense de Madrid. This was at the time that Velazquez was a Professor there. In this direction Vega and Zhu are looking at the connection of the BF with some mathematical models of the aortic valve of the human heart. The necessary numerical simulations will be done by De la Hoz.

Regarding line **(B)** we maintain S. Gutiérrez, C. E. Kenig and G. Ponce as external collaborators. Gutiérrez in connection with the evolution of vortex filaments, and Kenig and Ponce for the work on uncertainty principle and its relation with control and inverse problems and uniqueness of PDE's. A. García and M. Zubeldia are two former students of L. Vega that have been hired in Helsinki within L. Paivarinta's group of Inverse Problems. That field of research is a natural continuation of the work our group has been doing on Helmhotlz equation and therefore it can be considered as a new direction with respect to our previous project.

Regarding **(C)**, De la Hoz and Vadillo have strengthen their relation during the period 06/12 as can be deduced form the number of their joint papers and in the common project they are making in this proposal. J. Rivas will continue her work with the group of A. Buffa in Pavia. J. Aguirre is interested in renewing his relation with Rivas after the period he has been the Director of "Acceso" of the UPV/EHU (for this reason we have considered him as 1/2 EJC). De la Hoz and Vega will continue the research on the vortex filament equation. More concretely the use of numerical methods to study the evolution of closed polygons is crucial. C. Gorria and Y. Gadidei (as an external member) have left our group to become part of a new proposal that has D. Pardo as Principal Investigator. In any case we will continue the collaboration with Gorria that we expect to extended to the rest of that group.

Next we proceed to explain the concrete aims and objectives of our proposal.

(A) MATHEMATICAL PHYSICS.

Rigorous quantum many-body theory is a notoriously difficult subject. The hurdles that have to be overcomed in order to arrive at important new mathematical results involves many different fields of mathematics such as probability theory, operator algebras, differential equations or functional analysis. Also the evolution and interaction of vortex filaments are crucial aspects in the understanding of turbulence.

(A1) Kinetic Equations. (M. Escobedo and J.J.L. Velazquez)

Those equations are introduced as approximations to describe very large systems of particles. The Boltzmann equation for dilute classical gases is perhaps one of the best known examples. Our main interest in this project is focused on two examples of Boltzmann type kinetic equations: the Boltzmann equation for bosons and coagulation equations.These equations are two particularly interesting examples due to the existence of singular solutions. These solutions are related to two physical phenomena that have been observed experimentally: gelation and Bose Einstein condensation. One of the purposes of our work is to precise these relations in rigorous mathematical terms.

We have already obtained several partial results about these two equations that may be very roughly described as:

(a) Gelation in finite time for coagulation equations
(b) Existence of non zero flux solutions for the Boltzmann equation for bosons.
(c) Existence of non zero flux solutions for coagulation equations.

Our main purpose in this new project is twofold. On the one hand, we will study the approximation of the finite system of linear equations, describing the finite system of interacting particles by the corresponding kinetic equation. This was performed for the Boltzmann equation for classical particles in several works by Lanford (1975) and Illner and Pulvirenti (1989). It has been done more recently for different systems of particles. The problem for the Boltzmann equation for bosons is difficult. However, is a bit simpler for the coagulation equation. In fact, we have already made some progress for coagulation systems of particles that do not undergo gelation. A related question is to understand the mechanism by which the limiting kinetic equation is regularized in order to avoid the singularity formation. On the other hand, we will continue our work about singular solutions of the Boltzmann equation for bosons and their relation with the Bose Einstein condensation. In particular, this requires to understand the equation for general measure valued functions. Another interesting aspect is the relation of the Boltzmann equation for bosons with the kinetic equations appearing in the so-called weak turbulence theory that applies to classical waves.

(A2) Mathematical Foundations of Statistical Mechanics. (J.B. Bru)

This issue is at the interface between mathematics (C*-algebras, convex analysis, game theory, variational problems, functional analysis and dynamical systems) and physics (superconductors, lasers). The aim is to develop and apply a new systematic method to investigate equilibrium/steady states (and thus all physical properties) of a macroscopic physical system. Examples of such systems are given by extensions of the BCS theory of superconductivity, the Hubbard model or laser physics. This program will start from our monograph of 163 pages to appear in Memoirs of the AMS, which explains the structure of equilibrium states of long range interactions (cf. the BCS superconductivity theory) by using the algebraic formulation of statistical mechanics. This research direction is also related to an old open problem in mathematical physics, first addressed by Ginibre in 1968, about the validity of the celebrated Bogoliubov approximation (widely used in quantum statistical physics) on the level of states.

(A3) Fluid Flow in Porous Media. (C. Cuesta and J.J.L. Velázquez)

The research will be focused on some mathematical problems that aim to understand various aspects of fluid flow in porous media. Traditionally, the mathematical study of such flows concentrates on the qualitative study of the semi-empirical macroscopic descriptions of flows in porous media, typically encoded as systems of nonlinear PDEs (these are 'averaged' equations describing the flow in a representative elementary volume), that are widely used in the engineering community, both in agricultural and oil extraction applications. Most interestingly, there are rigorous results concerning the derivation of macroscopic laws from the flows taking place at the pore scale (see e.g. [6]). Not surprisingly, such attends are restricted to single-phase flow or toy-models mimicking essential features of multi-phase flow (e.g. [5]): when two or more immiscible fluids are present, there are, apart from the tortuous geometry of the porous material, additional complications due to the presence of fluid-fluid and/or fluid-fluid-solid interfaces.

Understanding some properties of two-phase flows at the pore scale in order to deduce macroscopic laws of such systems is a daunting task. Our aim is to undertake small steps towards this end. Typically, the preliminary studies that we are carrying out are concentrated on gas-liquid flows where the action of gravity is crucial.

The first step is the study of thin-films going down a surface due to both the actions of gravity (pushing the fluid downwards) and the curvature of the substrate (in regions of negative curvature the fluid would tend to accumulate, whereas in regions of positive curvature the fluid would spread away and eventually break into drops). To date, we have been concentrated in the two-dimensional case and investigating in which situations the model can be simplified by employing a lubrication approximation for very viscous flows. The analysis reveals the nature of the regions where the liquid is accumulated in the presence of capillary and gravity forces (see [1], [2]). Further investigation is needed to extend the results to the three dimensional case and to the study of scenarios where the lubrication approximation cannot be applied.

Another step is an attempt to upscale a simple two dimensional porous medium, where grains of the same radius are uniformly distributed in space. We assume that part of the medium is filled up by a liquid and investigate the structure of the interface as a whole. We first consider a steady situation and analyse different regimes depending on the relative dominance of gravity and of surface tension. The aim is to derive global properties such as global changes in pressure (see [3]). This is motivated by a well-known theory that predicts hysteresis in two-phase flow systems, see e.g. [4]. Further work will include incorporating this preliminary stochastic analysis in an evolution model.

In addition to the analytical investigations of these problems, one would require to perform numerical simulations that can sed light into the validity of the proposed models and of the behaviour of the found solutions. For instance, performing dynamic stability studies of special solutions of the evolution problems seems natural. Such analysis will benefit from the collaboration with the group members Fernando Vadillo and Francisco de La Hoz, due to the demonstrated expertise in scientific computing (see list of publications).

[1] C.M. Cuesta and J.J.L. Velázquez, "Analysis of oscillations in a drainage equation", SIAM J. Math. Anal. 44, pp. 1588-1616 (2012).

[2] C.M. Cuesta and J.J.L. Velázquez, "Fluid accumulation in thin-film flows driven by surface tension and gravity (I): Rigorous analysis of a drainage equation", arXiv:1107.5917.

[3] M. Calle, C.M. Cuesta and J.J.L. Velázquez, "Interfaces determined by capillary forces and gravity in porous media", in preparation.

[4] K. J. Maloy, L. Furuberg, J. Feder, T. Jossang, "Dynamics of slow drainage in Porous Media", Physical Review Letters, 68, 14, 1992.

[5] Schweizer, Ben, "Laws for the capillary pressure in a deterministic model for fronts in porous media", SIAM Journal on Mathematical Analysis, 36, no. 5, 1489--1521, (2005).

[6] L. Tartar, "Incompressible fluid flow in a porous medium - convergence of the homogenization process", Non-homogeneous Media and Vibration Theory, Lecture Notes in Physics 127, Springer Verlag, Berlin, 1980, pp. 368-377.

(A4) Vortex Filaments. (F. De la Hoz, S. Gutiérrez, and L. Vega)

This line of research was started with the paper by Gutiérrez, Rivas, and Vega [1] and the one by Gutiérrrez and Vega [2], where selfsimilar solutions [1] and almost selfsimilar solutions with the shape of logarithmic spirals [2] of the BF were constructed and characterized. The stability of the selfsimilar solutions has been the object of the papers [4-7] and we think that this question will be finished within two years. One of the byproducts that has been obtained in the stability analysis is the existence of some unexpected dispersive breakdowns for solutions of the one dimensional, either focusing or defocusing, cubic non-linear Schrödinger equation. As far as we know, this type of breakdown is completely new in the field of dispersive equations.To what extent this is a common feature for other dispersive equations is a natural question that we will try to answer.

The stability analysis of the solutions of [2] has been started in [3] where a first non-trivial but small step has been done. In particular, we have found that there is a particular class of logarithmic spirals where the analysis is much easier than for the rest. We will start looking at them. In order to do this we shall need to answer some questions in the field of harmonic analysis, more concretely, to obtain some estimates of oscillatory integrals. This creates a link between the topics **(A)** and **(B)** of this project.

[1] S. Gutiérrez, J. Rivas, L. Vega, "Formation of singularities and self-similar vortex motion under the localized induction approximation", Comm. Partial Differential Equations 28 (2003), no. 5-6, 927–968.

[2] S. Gutierrez, L. Vega, "Self-similar solutions of the localized induction approximation: singularity formation", Nonlinearity 17 (2004), no. 6, 2091–2136.

[3] S. Gutierrez, L. Vega, "On the stability of self-similar solutions of 1D cubic Schrödinger equations", arXiv:1103.5403, to appear in Nonlinearity.

[4] V. Banica, L. Vega, "On the Dirac delta as initial condition for nonlinear Schrödinger equations", Ann. Inst. H. Poincaré Anal. Non Linéaire 25 (2008), no. 4, 697–711.

[5] V. Banica, L. Vega, "On the stability of a singular vortex dynamics", Comm. Math. Phys. 286 (2009), no. 2, 593–627.

[6] V. Banica, L. Vega, "Scattering for 1D cubic NLS and singular vortex dynamics", J. Eur. Math. Soc. (JEMS) 14 (2012), no. 1, 209–253.

[7] V. Banica, L. Vega, "Stability of the selfsimilar dynamics of a vortex filament", arXiv:1202.1106.

(A5) Breathers. (M. A. Alejo and L. Vega)

A related question to the stability of vortex filaments is the one of the stability of breathers. The breathers can be understood as non-linear wave packets or oscillatory pulses that do not disperse. A breakthrough in this direction has been recently achieved by Alejo and Muñoz in [1]. This work opens many questions. One of them is related to the degenerate situation when the frequency of the breather becomes close to zero. Some numerical experiments done by Alejo, Gorria, and Vega [2] suggest the existence of some inestability in this regime. We propose to study this issue in collaboration with C. Muñoz. This collaboration was started when Muñoz came as Visiting Professor to our department thanks to the previous grant 06/12.

Another question that we want to study is related to the stability of the models where breathers appear. So far, they have been found in the Sine-Gordon equation and in the Korteweg-de Vries (KdV) equation. In the KdV setting, they are known to exist just in the so-called modified KdV-Gardner equation. Our purpose is to prove that this is, in fact, the only possibility. In other words, that any other model that falls within the class of KdV equations does not have breathers. We think that the ideas developed in [3] will be useful for this purpose.

[1] M. A. Alejo, C. Muñoz, "Nonlinear stability of mKdV breathers", arXiv:1206.3157.

[2] C. Gorria, M. A. Alejo, L. Vega, "Discrete conservation laws and the convergence of long time simulations of the mKdV equation", arXiv:1109.6028. To appear in J. of Computational Phys.

[3] C. E. Kenig, G. Ponce, L. Vega, "Lower Bounds for Non-Trivial Traveling Wave Solutions of Equations of KdV Type", arXiv:1112.3505. To appear in Nonlinearity.

(A6) Materials Science. (P. Zhu)

Materials science has emerged as one of the central pillars of the modern physical sciences. Modeling serves as the basis for an understanding of materials and their response to external stimuli, and phenomenological models may be entirely satisfactory if the goal is to test the response of a given material to casting in different shapes. There are two main types of models: sharp interface model and phase field model. Two mechanisms of phase transitions can be distinguished: the diffusion dominated and diffusionless (also called martensitic or structural) phase transitions. Starting from sharp interface models, with the help of the second law of thermodynamics and a formula of configurational forces, by analogy with the famous Allen-Cahn and Cahn-Hilliard models, Alber and Zhu formulate phase field models **(a)** and **(b)**, see [1-3]. We will study the two phenomenological models.

Model **(a)**: A model for martensitic phase changes driven by configurational forces in e.g., shape memory alloys: a second-order degenerate nonlinear parabolic equation coupled to a linear elasticity system. The mobility in this parabolic equation depends on the gradient of order parameter, while the mobility of the Allen-Cahn model depends only on order parameter. In this case the order parameter is not conserved.

Model **(b)**: A model for motion of grain boundaries (an interface motion by interface diffusion, an important application is sintering): a nonlinear equation of the Cahn-Hilliard type (however the mobility is a nonlinear, non-smooth function of the gradient of order parameter) coupled to a linear elasticity system. In this case, the order parameter is conserved.

The models are of elliptic-parabolic type and the parabolic equation is degenerate due to the non-smooth gradient term. Therefore, up to now we have obtained some results only for one-dimensional problems [1-3,5], except some formal results of asymptotic expansions [4,5].

For both models, we investigate: multi-dimensional problems, numerical simulations, the sharp interface limit as the thickness of interfacial region goes to zero. We can also investigate the geometry of the interfaces for sharp interface models. The possible tools may be viscosity solutions, geometric measure theory, etc.

Using the key idea for the formulation of models **(a)** and **(b)**, we will propose new models for materials with multi-components, for propagation of cracks, for evolution of dislocations, etc. We will also work on an initial-boundary value problem for a model that describes the fractal fiber architecture of aortic heart valve leaflets determined by mechanical equilibrium. The model is equivalent to the bi-normal flow (BF) equation. See [6]. And other problems related to the Schrödinger equation.

[1] H. Alber and P. Zhu (2006) SIAM J. Appl. Math. 66 No. 2, pp. 680–699.

[2] H. Alber and P. Zhu (2007) Arch. Rati. Mech. Anal. 185, pp.235–286.

[3] H. Alber and P. Zhu (2008) Proc. R. Soc. Edinburgh 138A, pp.923–955.

[4] H. Alber and P. Zhu (2011) Conti. Mech. Thermodyn. 23, pp.139–176. Online: Aug., 2010.

[5] P. Zhu (2012) J. Math. Anal. Appl., and some other manuscripts.

[6] C. Peskin and D. McQueen (1994) Amer. J. Physiol., 266 H319 – 28. (See also the references by V. Banica and L. Vega.

(B) MATHEMATICAL ANALYSIS AND PARTIAL DIFFERENTIAL EQUATIONS.

(B1) Linearized Kinetic Equations. (M. Escobedo and J.J.L. Velázquez)

In the study of some nonlinear kinetic equations with collision terms, it is very natural to study the linearized equation around one of the equilibriums. In several interesting cases, the equilibrium may be an unbounded function and this fact leads to linear integral operators defined in some principal value sense; coagulation-fragmentation equations and Boltzmann equations for bosons are two relevant examples. If one is interested in using these linear operators to solve the nonlinear equation in a perturbative way it is necessary to understand their continuity and regularizing properties. This has been partially done in the two examples cited above where some rather particular regularizing effect has been proved to hold. For the coagulation equation, this effect happens at infinity (see [1]), although for the Boltzmann equation for bosons it happens at finite time.

Reference [2] contains some partial results, and a deeper study of these two examples should be performed. On the other hand, there exist several examples of nonlinear kinetic equations with collision integrals that may give rise to very similar but slightly different situations. These questions will be part of our work in this project.

[1] M. Escobedo, J. J. L. Velázquez, "Local well posedness for a linear coagulation equation'', to appear Trans. AMS.

[2] M. Escobedo, S. Mischler, J. J. L. Velázquez, "Singular Solutions for the Uehling Uhlenbeck Equation'', Proc. Roy. Soc. Edinburgh, vol. 138A, (2008), pp. 67-107.

(B2) Uncertainty Principle. (L. Escauriaza, A. Fernández, C.E. Kenig, S. Montaner, G. Ponce, and L. Vega)

(B2a) There is an equivalent formulation of Hardy’s uncertainty principle using the heat equation instead of the Schrödinger equation: if a solution of a heat equation that at time zero is integrable and at time one decays faster than the free fundamental solution of the heat equation then it must be zero. Our approach to solve this problem for the Schrödinger evolution is based on the use of Carleman estimates and Log-convexity properties [1-6] and can be also extended to the diffusion equation. Nevertheless there is an extra and fundamental recursive argument that is rather delicate and that we have not been able to optimize, so we are still missing the sharp version of Hardy’s theorem in this scenario. We consider that this is a fundamental question that has to be solved.

(B2b) We want to extend the methods we have developed to prove Hardy´s result to other uncertainty principles as Paley-Wiener theory or the more recent ones of Benedicks [7] and Nazarov [8]. In particular the quantitative result of Nazarov seems very appealing and it is hard to know up to what extent it will be true in the more general scenario of Schrodinger equations with non-trivial linear or non-linear potentials. We have given some preliminary steps in this direction that suggest that within this general framework the results that can be expected should be weaker. Extra porperties of the equation, as for example that any regularity of the initial datum is preserved at all times, would allow to conclude the result in its full strength and, as a consequence, to give another proof of Nazarov's result. We think this is a widely open field of research of interaction between PDE's and harmonic analysis.

The PhD thesis of A. Fernández and S. Montaner will be on these topics.

[1] L. Escauriaza, C.E. Kenig, G. Ponce, L. Vega, "On Uniqueness Properties of Solutions of Schrödinger Equations", Commun. Part. Diff. Eq. 35, 12 (2006), pp. 1811-1823.

[2] L. Escauriaza, C.E. Kenig, G. Ponce, L. Vega, "Convexity Properties of Free Solutions of Schrödinger Equations with Gaussian Decay", Math. Res. Lett. 15, 5 (2008), pp. 957-971.

[3] L. Escauriaza, C.E. Kenig, G. Ponce, L. Vega, "Hardy's Uncertainty Principle, Convexity and Schrödinger Evolutions", J. Eur. Math. Soc. 10, 4 (2008), pp. 883-907.

[4] L. Escauriaza, C.E. Kenig, G. Ponce, L. Vega, "The Sharp Hardy Uncertainty Principle for Schrödinger Evolutions", Duke Math. J. 155, 1 (2010), pp. 163-187.

[5] M. Cowling, L. Escauriaza, C.E. Kenig, G. Ponce, L. Vega, "The Hardy Uncertainty Principle Revisited", Indiana U. Math. J. 59, 6 (2010), pp. 2007-2026.

[6] L. Escauriaza, C.E. Kenig, G. Ponce, L. Vega, "Uniqueness Properties of Solutions to Schrödinger Equations", Bull. (New Series) of the Amer. Math. Soc. 49 (2012), pp. 415-442.

[7] M. Benedicks, "On Fourier transforms of functions supported on sets of finite Lebesgue measure", J. Math. Anal. Appl. 106 (1985), pp. 180-183.

[8] F. L. Nazarov, "Local estimates for exponential polynomials and their applications to inequalities of the uncertainty principle type", (Russian) Algebra i Analiz 5 (1993), pp. 3-66; translation in St. Petersburg Math. J. 5 (1994), pp. 663-717.

(B3) Control Problems. (L. Escauriaza, C.E. Kenig, G. Ponce, and L. Vega)

(B3a) To establish the lack of null-controllability of the heat equation with controls acting over the lateral boundary over cones with opening angle greater than or equal to 90º. This result has been established for cones with opening angle larger than 180º in [1] and [2] and for cones with opening angle smaller than but close to 180º in [3]. In the later work the authors published and recognized the authorship of counterexamples by L. Escauriaza which show that when the opening angle of the cone is less than 90º there are nonzero and bounded caloric functions over the cone which vanish at any fixed future time. As a consequence of the last counterexamples, we also want to study whether the null-controllability of the heat equation with boundary controls is possible over cones with opening angle smaller than 90º.

(B3b) Recently, C. Muñoz has proved the result of approximate controlability for the Korteweg-de Vries (KdV) equation in the full line (see [4]). This has been an open question that has interested Coron among others experts in Control Theory. We want to prove that the controlability can not be exact but it has to be approximate, as proved by Muñoz. The key idea is to prove a version of Paley-Wiener theorem for the difference of two solutions of KdV in the spirit of what we have obtained in the non-linear Schrödinger setting. The main difficulty comes from the lack of the conformal transformation. We think that this obstruction will be bypassed by making use of an appropriate Carleman estimate.

[1] L. Escauriaza, G. Seregin, V. Svêrák, "Backward uniqueness for parabolic equations", Arch. Rational Mech. Anal. 169 (2003), pp. 147-157.

[2] L. Escauriaza, G. Seregin, V. Svêrák, "Backward uniqueness for the heat operator in half-space", St. Petersburg Math. J. 15, 1 (2004) pp. 139-148.

[3] L. Li, V. Svêrák, "Backward uniqueness for the heat equation in cones", Commun. Part. Diff. Eqtns. 37, 8 (2012) DOI:10.1080/03605302.2011.635323.

[4] C. Muñoz, "On approximate controllability of generalized KdV solitons", arXiv:1206.2430.

(B4) Carleman Weights, Uniqueness, and Inverse Problems. (L. Escauriaza, A. García, and M. Zubeldia)

(B4a) To understand and to find the suitable and proper Carleman inequalities which would yield results on strong unique continuation within characteristic hyper-planes for solutions to one-dimensional fourth order parabolic equations and all along the lines of the works for parabolic second order equations within [1-5].

(B4b) Concerning the issue of reconstructing a conductivity from boundary measurements the mathematical theory developed in the last years tries to understand the problem of dealing with less regular conductivities and rough domains as well. In the spirit of the techniques used for the uniqueness of the inverse boundary value problem, we try to extend the existing results for regular conductivities and regular domains to the case of conductivities with low regularity and domains with boundary without any smoothness, in accordance with what is observed in the physics. Reference [6] is a first result that we have obtained in this direction.

[1] L. Escauriaza, F. J. Fernández, "Unique continuation for parabolic operators", Arkiv för Matematik, 41, 1 (2003) pp. 35-60.

[2] L. Escauriaza, F.J. Fernández, S. Vessella, "Doubling properties of caloric functions", Applicable Analysis 85, 1-3 (2006), pp. 205-223.

[3] L. Escauriaza, L. Vega, "Carleman inequalities and the heat operator", II. Indiana Univ. Math. J. 50, 3 (2001), pp. 1149–1169.

[4] L. Escauriaza, "Carleman inequalities and the heat operator", Duke Math. J. 104, 1 (2000), pp. 113–127.

[5] H. Koch, D. Tataru, "Carleman Estimates and Unique Continuation for Second Order Parabolic Equations with Nonsmooth Coefficients", Comm. in Partial Diff. Eqtn. 34, 4 (2009), pp. 305-366.

[6] P. Caro, A. García, J. M. Reyes, "Stability of the Calderón problem with less regular conductivities", to appear in J. Diff. Equations.

(B5) Laplace Equation in Lipschitz Domains. (L. Escauriaza and A. Mas).

(B5a) The potential methods have been a successful approach to solve the Dirichlet or Neumann problems for the Laplace equation and for the lateral Dirichlet or Neumann problems with zero initial data for the Heat equation over space-time cylinders. More precisely, the invertibility of the boundary operators associated to the traces of the doube and single layer potentials of suitable densities on the lateral boundary (See [1-3] for the Laplace equation and [4,5] for the Heat equation). A transposition argument shows that the lateral Dirichlet problem with zero initial data for the Schrödinger evolution has a suitable unique solution. We want to show that the latter methods can also be applied to find the corresponding solutions for the free Schrödinger evolution and, probably, to find new regularity properties of the weak solutions constructed by transposition.

(B5b) An interesting open question where there have been no improvements since 1997 consists on finding the less regularity conditions on Lipschitz domains so that there are nontrivial and non-constant harmonic functions which vanish identically on a neighborhood of a boundary point and whose gradient vanishes on a set of positive surface measure within the neighborhood. The same question remains open when the zero Dirichlet type condition is replaced by a zero Neumann type condition. The first question has been settled for convex domains or domains whose unit normal vector has a Hölder modulus of continuity and the second for domains whose unit normal vector has a Lipschitz modulus of continuity in [6] and [7]. We plan to continue studying these matters: it would be a great mathematical accomplishment to lower the conditions on the regularity to domains whose unit normal vector varies continuously for the first case and for domains with unit normal vector with a Hölder modulus of continuity.

[1] O. D. Kellogg, "Foundations of Potential Theory", New York: Dover publications (1954).

[2] E. Fabes, M. Jodeit, N. Riviére, "Potential techniques for boundary value problems on C1 domains", Acta Math. 141 (1978), 165-186.

[3] G. C. Verchota, "Layer potentials and regularity for the Dirichlet problem for Laplace's equation in Lipschitz domains", J. of Funct. Anal. 69 (1984) pp. 572-611.

[4] R. M. Brown, "The initial-Neumann problem for the heat equation in Lipschitz cylinders", Trans. Amer. Math. Soc. 320, 1 (1990), pp. 1–52.

[5] R.M. Brown, "The method of layer potentials for the heat equation in Lipschitz cylinders", Amer. J. Math. 111, 2 (1989), pp. 339–379.

[6] V. Adolfsson, L. Escauriaza, C.E. Kenig, "Convex Domains and Unique Continuation at the Boundary", Rev. Mat. Iberoam. 11, 3 (1995), pp. 513-525.

[7] V. Adolfsson, L. Escauriaza, "C^{1,alpha} Domains and Unique Continuation at the Boundary", Commun. Pur. Appl. Math. L (1997), pp. 935-969.

(B6) Weighted Estimates for Classical Operators. (J. Duoandikoetxea, A. Moyua and O. Oruetxebarria)

(B6a) Weighted estimates for the spherical maximal operator acting on radial functions were obtained by J. Duoandikoetxea, A. Moyua and O. Oruetxebarria, and were published in [1]. The relation between the spherical means and the solution of the wave equation in three dimensions suggests the possibility of applying similar techniques to obtain weighted estimates, at least for power weights, for radial solutions of the wave equation in any space dimension. It is worth noticing that in general even the range of unweighted estimates is not completely known. On the other hand, solving the question for the wave equation could be a first step to deal with other operators, like those of Grushin type, for instance.

(B6b) The characterization of the classes of weights for the Calderón operator as weights of Muckenhoupt type corresponding to a suitable maximal operator has been obtained by J. Duoandikoetxea, F. J. Martín-Reyes and S. Ombrosi (paper accepted in Indiana Univ. Math. J.). Apart from the characterization itself, an interesting feature of such weights is that they do not satisfy some of the typical properties of the usual Ap weights (for instance, the reverse Hölder inequality can fail). We plan to study the classes of pairs of weights for the maximal operator and their relation with the weights for the Calderón operator. In this case we shall consider not only weights absolutely continuous with respect to Lebesgue measure, but also weights with a singular part.

In the case of the weights associated to the Hardy-Littlewood maximal operator, there exist several characterizations of the "A infinity" class. We know that, for the classes of weights studied in the above mentioned work, such characterizations are not equivalent. It seems interesting to determine the relations among the different characterizations in this context.

(B6c) The inequalities with Ap weights for several classical operators have been studied in the last decade from a new point of view: the sharp bound of the norm of the operator in terms of the Ap-constant of the weight. Since the classical proofs do not give sharp bounds, new techniques have been developed. A recent paper by K. Li and W. Sun (J. Math. Anal. Appl., 2012) shows sharp bounds in the range (1,3] for the maximal Bochner-Riesz operator in the critical index. The question for values of p greater than 3 remains open and we plan to deal with this problem in the future, as well as with the sharp bounds of the Bochner-Riesz operator itself (not the maximal one).

[1] J. Math. Anal. Appl. 387 (2012), pp. 655-666.

(B7) Dirac Equation. (N. Arrizabalaga, J. Duoandikoextxea, A. Mas, and L. Vega)

(B7a) Based on Arrizabalaga's thesis (see [1]), we want to give a new proof (of a more quantitative nature) of the fact that Dirac operators with critical perturbations of Coulomb type are selfadjoint. We want to focus in obtaining weighted inequalties for the Dirac operator with and without mass. This makes a clear link with **(B7b)** below. Special enphasis will be made in obtaining the sharp constants of these inequalities and a full description of the domain of definition of the perturbed Dirac operator, something that, as far as we know, is missing in the literature. Once this is done, we want to extend these ideas to obtain resolvent estimates for Dirac operators, as done in Zubeldia's thesis for the Laplace equation. Finally, the time dependent setting will be also studied.

(B7b) We want to extend the ideas explained in **(B7a)** for critical Coulomb perturbations to hamiltonians given by potentials which are singular measures on "reasonable" sets of Hausdorff dimension 2. We have to recall that the Dirac operator can be seen as an extension to higher dimensions of the Cauchy operator. As far as we know, only the case when the reasonable set is the sphere has been considered (see [2]) and it is done by using (in a very strong way) the spherical coordinates to reduce the question to a Fourier analysis problem in the 2-sphere. We consider that this is a complete new line of research.

[1] N. Arrizabalaga, "Distinguished self-adjoint extensions of Dirac operators via Hardy-Dirac inequalities", J. Math. Phys. 52 (2011), no. 9, 092301, 14 pp.

[2] J. Dittrich, P. Exner, and P. Seba, "Dirac operators with a spherically symmetric d-shell interaction", J. Math. Phys. 30 (1989), pp 2875-2882.

(C) NUMERICAL ANALYSIS.

(C1) NURBS. (J. Rivas and J. Aguirre)

In [1], approximation properties of some NURBS (NonUniform Rational B-Splines) spaces are analysed. More precisely, some estimates in two dimensions are proved. These estimates are explicit in the three discretization parameters: mesh size, h; degree of the piecewise polynomials, p; and regularity, k. However, these estimates are only valid for values of k less than or equal to (p+1)/2, and the problem remains open for k between (p+1)/2 and p.

Judith Rivas, in collaboration with Dr. Annalisa Buffa and her team, will continue working on the achievement of hpk-estimates for NURBS. The method developed in [1] cannot be applied for values of k greater than (p+1)/2, therefore a new approach must be searched.

The starting point will be to analyse the approximation properties of one-dimensional periodic splines. In this particular case, we expect some techniques of harmonic analysis may be useful due to the fact that when the degree p is big enough, the piecewise polynomials are in fact global polynomials. Once hpk-estimates for periodic splines are obtained, the aim is to generalise them to one-dimensional B-splines. The third step will be to consider B-splines in several dimensions. The final challenge is to obtain estimates explicit in h, p and k for NURBS spaces in several dimensions.

[1] Some estimates for h– p–k-refinement in Isogeometric Analysis, L. Beirão da Veiga, A. Buffa, J. Rivas, G. Sangalli, Numer. Math. (2011) 118:271–305

(C2) Differentiation Matrices and Matrix Differential Equations. (De la Hoz and Vadillo)

The concept of differentiation matrices has been shown to be a very powerful tool when computing numerical solutions of partial differential equations with MATLAB, in the frame of pseudo-spectral methods [1,2]. This idea, combined with the theory of matrix differential equations [3,4], may yield effective and reliable algorithms for tough problems in a natural and intuitive way [5]. We propose to use this approach to implement efficient numerical integrators for the spherical shallow water equation. This equation contains the essential wave propagation mechanisms found in more complete models and has been used to test numerical methods that are applied to atmospheric models.

[1] L.N. Trefethen, "Spectral Methods in MATLAB", SIAM, 2000.

[2] J.A.C. Weideman, S.C. Reddy, "A MATLAB Differentiation Matrix Suite", ACM Trans. on Math. Software, 2000 (26), pp. 465-519.

[3] R. Bellman, "Introduction to Matrix Analysis", 2nd Edition, SIAM, 1997.

[4] N.J. Higham, "Functions of Matrices", Theory and Computation, SIAM, 2008.

[5] F. de la Hoz and F. Vadillo, "Numerical simulation of the N-dimensional sine-Gordon equation via operational matrices", Comput. Phys. Commun., 2012 (183), pp. 864-879.

(C3) Numerical Analysis of the Backward Kolmogorov Differential Equation for Stochastic Models. (De la Hoz and Vadillo)

A stochastic model is a system of stochastic differential equations with a deterministic part plus another stochastic one. Although there have been numerous theoretical studies on the evolution, persistence and extinction in deterministic population models, the stochastic models are more complicated and have quite different persistence and extinction behaviors [1,2]. Therefore, an analysis of their persistence time and mean persistence-time is necessary. One of the most important applications of those models is the obtention of estimates for stochastic volatility models in finance [3] and stochastic epidemic models [4].

The mean persistence-time of the stochastic models satisfies the backward Kolmogorov differential equation, a linear second-order partial differential equation with variable coefficients for which is impossible to know any exact solutions. We propose to compute it through numerical approximations, by means of a finite-element method [5].

[1] L.J.S. Allen, "An Introduction to Stochastic Processes with Applications to Biology", Person Prentice Hall, 2003.

[2] E. Allen, "Modeling with Itô Stochastic Differential Equations", Springer, 2007.

[3] C. Kahl, "Modeling and Simulation of Stochastic Volatility in Finance", Dissertation.com, Boca Raton, Florida, 2007, ISBN: 1-58112-383-3.

[4] L.J.S. Allen and N. Kirupaharan, "Asymptotic dynamics of deterministic and stochastic epidemic models with multiple pathogens", Int. J. Numer. Anal. Model., 2005 (2), pp. 329-344.

[5] F. de la Hoz and F. Vadillo, "A mean extinction-time estimate for a stochastic Lotka-Volterra predator-prey model", Appl. Math. Comput., 2012 (219), pp. 170-179.

INTERNATIONALIZATION:

Our purpose is to keep publishing at the same rate we have been doing during the last six years and to try to improve the quality and impact of our papers. We also want to increase the number of small workshops organized by us. We prefer small meetings because we think they are much more effective than the large ones. In collaboration with D. Lannes (Ecole Normale Superieur, Paris) we have organized two meetings, one in Bayonne in January 2011 and the second one in Bilbao in January 2012. The next one will be organized in Rome in January 2013